Brain Surface Conformal Parameterization with Algebraic Functions

[1,2]Yalin Wang , [3]Xianfeng Gu, [1]Tony F. Chan,
[2]Arthur W. Toga, [2]Paul M. Thompson, [4]Shing-Tung Yau
[1]Department of Mathematics, UCLA, Los Angeles, CA, USA.
[2]Laboratory of Neuro Imaging, Brain Mapping Division, Department of Neurology, UCLA School of Medicine, Los Angeles, CA, USA.
[3]Computer Science Department, SUNY at Stony Brook, Stony Brook, NY 11794, USA.
[4]Mathematics Department, Harvard University, Cambridge, MA 02138, USA

Objective: Our previous work conformally parameterizes various brain anatomical surfaces with several planar parallelograms, using a holomorphic flow segmentation algorithm. However, the parameterization has some distortion in the areas surrounding the zero points. This makes it less accurate for matching two surfaces in zero point areas. Here we propose a new surface conformal parameterization method based on algebraic functions. We find a planar conformal parameterization without any singularities by solving the *Yamabe equation* with the *Ricci flow*. This provides a simple way to match landmarks exactly acrossdifferent brain anatomical surfaces.

Methods: Suppose *M* is a genus zero surface with *n+1* holes, *N* is a n-hole punctured disk, *ø:M→N* is a conformal map between them, *g* is the metric on *M.* From the theory of complex geometry, the problem to find a conformal map *ø:M→N* is equivalent to find a function *u:M→R, such that* $\hat{g} = e^{2u} g$ is the metric on *M* induced by *ø,* which satisfies the Yamabe equation

$$\begin{cases} \tilde{K} = 0 \\ \Delta u - K + e^{2u}\tilde{K} = 0 \\ k_{\tilde{g}|\partial M} = const \end{cases}$$

We can solve this equation with the Ricci flow method,

$$\frac{du(t)}{dt} = \tilde{K} - K(t)$$

Basically, with the Ricci flow method, the domain surface *M* is conformally deformed such that the Gaussian curvature of the interior points are zeros everywhere and the boundaries become circles.

Acknowledgement: This work is supported by grants from NIH under contracts MH65166 and U54 RR021813

Results:

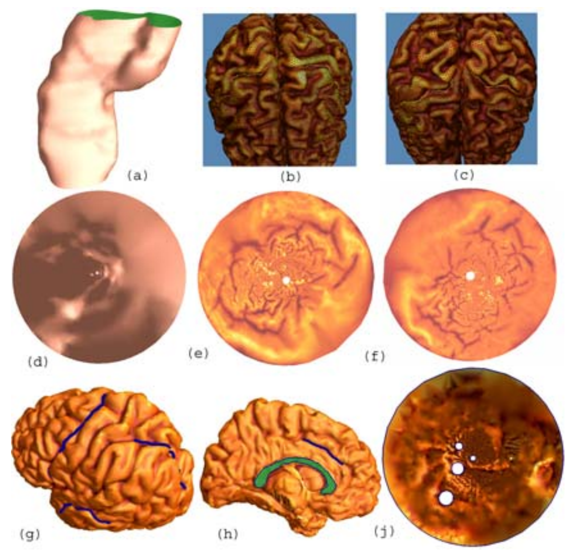

Figure 1. Illustrates conformal surface parameterization. (a) shows the front view of a hippocampal surface and (d) shows its conformal map to a 1-hole disk. (b) and © are two cerebral cortical surfaces. Two central sulci area labeled as yellow curves on each of them. After cutting along the landmark curves, each of them is conformally mapped to a 1-hole disk ((e) and (f)) (g)-(j) show a conformal map from a left hemisphere cortex with 5 labeled landmarks to a 4-hole disk.

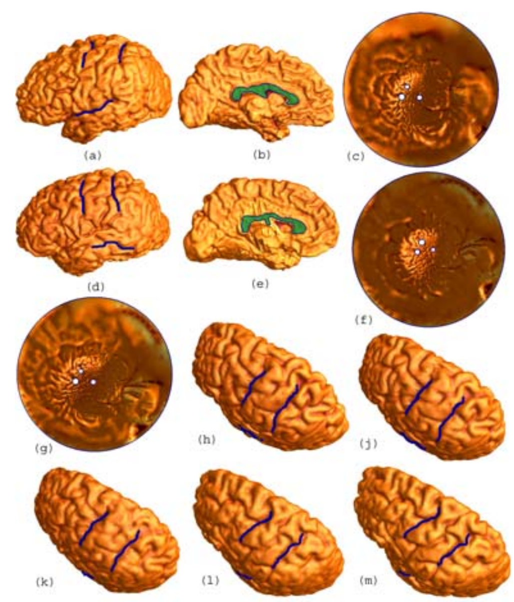

Figure 2. Illustrates direct surface matching between two different cerebral cortical surface while explicitly matching landmark curves. (a)-(b) show a left cerebral cortex with four labeled landmarks and (c) shows its conformal map to a 3-hole disk. (d)-(f) show another left hemisphere model and its conformal map to a 3-hole disk. (g) is the parameterization of surface (d)-(e) after a constrained harmonic map from (f) to (c) is built. (h)-(m) show a morphing sequence from surface (a)-(b) to surface (d)-(e). Although the cortical surface shape changes considerably, the relative positions of the selected landmark curves do not change.