Hippocampal Surface Analysis using Spherical Harmonic Functions Applied to Surface Conformal Mapping

[1]Boris Gutman, [1,2]Yalin Wang , [1]Lok Ming Lui, [1]Tony F. Chan, [2]Paul M. Thompson

[1]Department of Mathematics, UCLA, Los Angeles, CA, USA.

[2]Laboratory of Neuro Imaging, Brain Mapping Division, Department of Neurology, UCLA School of Medicine, Los Angeles, CA, USA.

Objective:

Using spherical harmonics of an inverse conformal map, we compared hippocampal surfaces of sixteen Alzheimer (AD) and fourteen control subjects. Use of spherical harmonics as initial shape descriptors is advantageous to methods involving neuroanatomical templates in that the transform is independent of any population-based averages and that it describes global shape features in addition to locally detailed features.

Methods:

Conformal mapping onto the 2-sphere:

Suppose *M* is a closed genus zero surface which is represented by a triangulation mesh embededd in R^3*,* first, we can compute a conformal mapping $f:M \rightarrow S^2$ by minimizing harmonic energy,

$$E(f) = \int_M \|\nabla f\|^2 \, d\sigma_M$$

The discrete harmonic energy is

$$E(f) = \sum_{[u,v]\in M} k_{uv} \|f(u) - f(v)\|^2 \qquad k_{uv} = \tfrac{1}{2}(\cot\alpha + \cot\beta)$$

The discrete Laplacian is

$$\Delta f(u) = \sum_{[u,v]\in M} k_{uv}(f(u) - f(v))$$

Spherical harmonic analysis applied to conforaml mapping

Let $\bar{f} : R^3 \supseteq M \rightarrow S^2$ be a conformal homeomorphism defined discretely. Let $\bar{f}^{-1} : S^2 \rightarrow R^3$ be the inverse map from the sphere onto the hippocampal surface, defined by the isomorphic property of the homeomorphism. We project the inverse of the discrete conformal map onto a finite-dimensional subspace of. The result is a set of vector spherical harmonic coefficients in

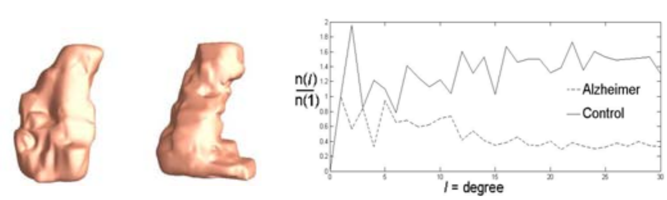
$$\{\hat{\bar{f}}^{-1}(l,m) = (\hat{f}_1^{-1}(l,m), \hat{f}_2^{-1}(l,m), \hat{f}_3^{-1}(l,m)) \mid |m| \le l < B\}$$

where *B* is the bandwidth.

The spherical harmonic descriptors should be invariant to rotation, as each of the inner sums is theoretically invariant. It is also possible to make the coefficients translationally invariant by simply disregarding the degree-zero coefficient, which is alone responsible for translation. Now, object surface comparison is possible directly in the simplified spectrum domain without further registration.

Results:

we compared hippocampal surfaces of sixteen Alzheimer (AD) and fourteen control subjects. A 3-way ANOVA was run on normalized descriptors with hemisphere, diagnosis and frequency as factors. Only descriptors up to frequency 15 were considered. Most notably, the test yielded significant results for diagnosis (df=1, F=4.061, α=.044), and interaction between diagnosis and hemisphere (df=1, F=7.520, α=.006). Other significant factors include hemisphere (df=1, F=6.429, α=.011), and interaction between frequency and hemisphere (df=14, F=2.694, α=.001 . Further, the specific frequencies 2,6, 8,12 and 15 when taken alone, generated a significant interaction effect between diagnosis and hemisphere, so as to correctly predict 21/30 subjects' diagnosis.



(a) (b) (c)

Figure 1. (a) is a left hippocampal surface of a female Alzheimer subject and (b) is a left hippocampal surface of a female control subject. (c) is plots of normalized descriptors for two left hippocampi: higher relative magnitude in lower degree harmonics of the solid line indicates a greater presence of low-frequency curves in the control hippomcampal surface.

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