Theoretical Models relating
Angular and Spatial Resolution Trade-offs in HARDI

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Introduction: High angular resolution diffusion imaging (HARDI) has been proposed to resolve complex diffusion probability density functions in the brain (such as fiber crossings and intermixing of white matter tracts). Traditional DTMRI cannot resolve these fiber crossings, and DTMRI-derived measures, such as fractional anisotropy (FA), are inaccurate in the 30-50% of the brain's voxels where fibers mix [1]. HARDI collects more gradient images than DTMRI, so if scan time is limited, there is a tradeoff between spatial and angular resolution. When scan duration is limited to reduce patient discomfort, higher angular resolution can make white matter fiber-tracking more accurate, but this also means lower spatial resolution, leading to a higher partial volume effect in the resulting data. For a fixed scan time, scans with higher spatial resolution will have less partial volume effect but lower angular resolution, which will worsen the accuracy of fiber tractography.

Methods: In MRI theory, SNR is proportional to voxel volume and to the square root of acquisition time (**Eq 1**, Fig 1), so SNR falls precipitously as spatial resolution increases. Thus, trade-offs between angular and spatial resolution must be established to obtain the best image quality in the least time. Previous studies have assessed how increasing the number of diffusion directions influences SNR for different DTMRI-derived measures [2, 3] and reconstruction errors in the principal eigenvector field, which is important for fiber tractography [4]. Even so, to our knowledge, no studies have examined the trade-off between spatial and angular resolution. Since angular and spatial resolution are approximately inversely related for acquisitions of fixed duration, we designed two experiments to model how angular resolution and spatial resolution affect HARDI individually. For randomly-generated tensors with eigenvalues in the normal range for human white matter, with known FA (FA_1) (**Eq 2**), we generated diffusion weighted signals (Eq. 3) for different angular resolutions (gradients are selected using Eq 4 [5]). We artificially added different levels of Rician noise to these diffusion-weighted signals, and re-estimated FA from the noisy data (FA_2. The absolute difference between FA_1 and FA_2, called delta(FA), was used to evaluate how spatial resolution affects errors in our simulated HARDI. This process was repeated 1,000,000 times. In the second experiment, as we know, the partial volume effect depends mostly on the object geometry, so we designed a hybrid experiment using experimental and simulated data. We first collected a real human HARDI scan (using 41 DWIs, 4 b0 scans, 2.5mm cubic voxels and b=1000 s/mm^2). We sub-sampled this data to create several new datasets with isotropic voxels of size 2.6-5.0mm, in 0.1mm increments. For each voxel size, we calculated FA, and correlated the results with FA from the original experimental data, yielding a correlation value, corr(FA).

Results: Figure 2 shows how the error in the FA estimates, delta(FA), changes (1) as SNR was varied, and (2) with different angular resolutions. **Figure 3** shows how corr(FA) changes with different voxel sizes. Based on these figures, data from larger voxels showed lower overall correlation with ground truth, but still achieved a relatively small absolute error (delta). We modeled this tradeoff and proposed a reasonable cost function to optimize scan quality for measuring FA (Equation 5), as a function of voxel size and scan time; FA_o was calculated using the lowest angular resolution, for example: 6 DWI plus one b0 scan for basic DTMRI; w is a weighting factor. Correlations between FA estimates and ground truth fell gradually as voxel size increased. At low SNR, higher angular resolution greatly improved the accuracy of FA estimates, but this mattered less when SNR was high.

Conclusions: These dependencies show the value of testing several DTMRI protocols to maximize measurement accuracy in a limited scan time.

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$$\text{SNR} \propto (\Delta x)(\Delta y)(\Delta z)\sqrt{\text{scanning time}} = (\text{voxel size})^3 \sqrt{\text{scanning time}} \qquad (1)$$

$$FA = \sqrt{\frac{3}{2}\left(\frac{(\lambda_1 - \hat{\lambda})^2 + (\lambda_2 - \hat{\lambda})^2 + (\lambda_3 - \hat{\lambda})^2}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}\right)} \qquad (2)$$

$$S(q) = S(0)\exp(-bq^T D q) = \exp(-bq^T D q) \qquad (3)$$

$$\begin{cases} g_z(n) = \dfrac{2n - N_e - 1}{N_e} \\[2mm] g_x = \cos\left(g_z\sqrt{N_e\pi}\right)\sqrt{1 - g_z^2} \\[2mm] g_y = \sqrt{1 - g_z^2 - g_x^2} \end{cases} \qquad (4)$$

$$\text{Cost function} \triangleq w * \log\left(\frac{(\text{Voxel Size}_x)^3 * \sqrt{\text{scanning time}_x}}{(\text{Voxel Size}_y)^3 * \sqrt{\text{scanning time}_y}}\right) + (1 - w) * \log\left(\text{corr}(FA_x, FA_0)\right) \qquad (5)$$

Figure 1.

Figure 2

Figure 3