HIPPOCAMPAL MORPHOMETRY STUDIED WITH BRAIN CONFORMAL MAPPING

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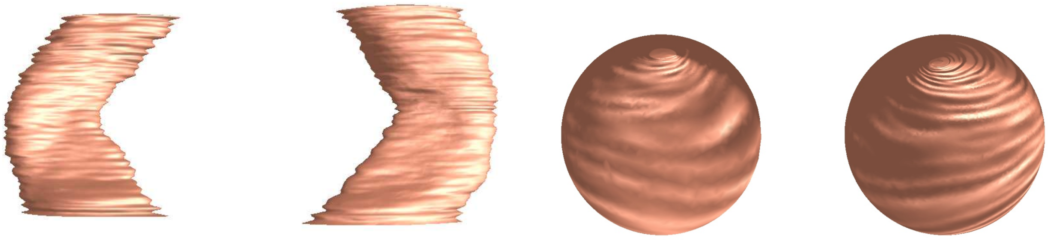

(a) (b) (c)

FIGURE 1. Illustrates the conformal mapping on hippocampal surfaces. (a) is the view, from above, of left hippocampus and right hippocampus; (b) is the result of conformally mapping the left hippocampus onto a sphere; (c) is the result of conformally mapping the right hippocampus onto a sphere. These mappings are used to induce a conformal coordinate system onto brain structures for cross-subject comparisons and 3D shape analysis.

There is growing interest in studying the brain structure abnormalities for the diagnosis and monitoring of brain diseases, such as schizophrenia and Alzheimer's disease. With a novel and rigorous brain conformal mapping technique, we propose a 3D shape representation, based on spherical harmonic functions, to analyze hippocampal shapes in a large database which consists of 80-90 hippocampus models derived from magnetic resonance images (MRI). These studies can reveal systematic patterns of hippocampal shape deformation in clinical research.

The hippocampus is a part of the brain located inside the temporal lobe (there are two hippocampi, one on each side of the brain). It is a part of limbic system and plays a role in memory and navigation. In Alzheimer's disease, the hippocampus is one of the first regions of the brain undergoing structural changes. With surface reconstruction tools, we can build geometric models of hippocampi from MRI volumes. Geometric surfaces are commonly represented as triangle meshes in the medical imaging field. We treat these surfaces as complex manifolds and compute their holomorphic differentials (conformal structures). Any genus zero

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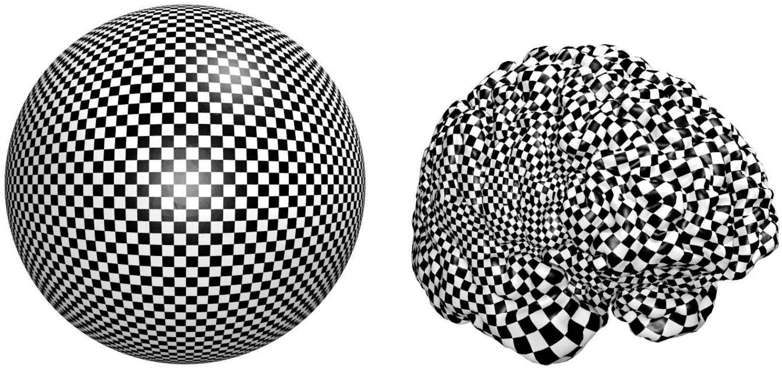
surface can be mapped conformally onto a sphere and any local portion thereof onto a disk. This mapping, a conformal equivalence, is one-to-one, onto, and angle-preserving. Moreover, the elements of the first fundamental form remain unchanged, except for a scaling factor. For this reason, conformal mappings are often described as being similarities in the small. The cortical and hippocampal surfaces of the brain are genus zero surfaces, so conformal mapping offers a convenient method to retain local geometric information, when mapping data between the surface and a sphere. For genus zero surfaces, conformal mapping can be obtained by minimizing the harmonic energy. To ensure the algorithm converges, constraints are added to ensure that there is a unique conformal map. Figure 1 shows a conformal mapping from the surfaces of the left and right hippocampus to a sphere. Figure 2 shows the right angles are optimally preserved for this conformal mapping, in this case computed for the brain surface (cerebral cortex).

A function $f : S^2 \to \mathbb{R}$ is called a spherical harmonic, if it is an eigenfunction of the Laplace-Beltrami operator, namely $\Delta f = \lambda f$, where λ is a constant. There is a countable set of spherical harmonics which form an orthonormal basis for $L^2(S^2)$. After the surface is mapped to a sphere, the surface parametrization can be expanded into a series of spherical harmonic functions, where shapes are represented as weighted sums of basis functions with varying frequencies (see G. Gerig et al. for related work). In the Fourier transformation, low frequencies represent a coarse representation of the signal in terms of waves with larger wavelength, whereas higher frequencies add more refined information. Similarly, the low order spherical harmonic functions represent coarse features of three-dimensional structures, and higher order functions add details of the objects' surfaces. Once the hippocampal surface is conformally mapped to S^2, the surface can be represented as three spherical functions using the fast spherical harmonic transformation. The coefficients of the different spherical harmonic functions are the shape descriptors of the surface.

Given the surface descriptors, we can then apply statistical analysis to a large data set. In current work, we are studying interclass and intraclass variations and building a statistical atlas. With this statistical atlas, automated image segmentation and applications to diagnosis of disease can be further explored.



(a) (b)

Figure 2. Illustrates the angle-preserving property of the conformal mapping. (a) Texture mapping of the sphere; (b) texture mapping of the brain. The right angles in (a) are well-preserved in (b).