Optimization of Conformal Parametrization using Landmarks

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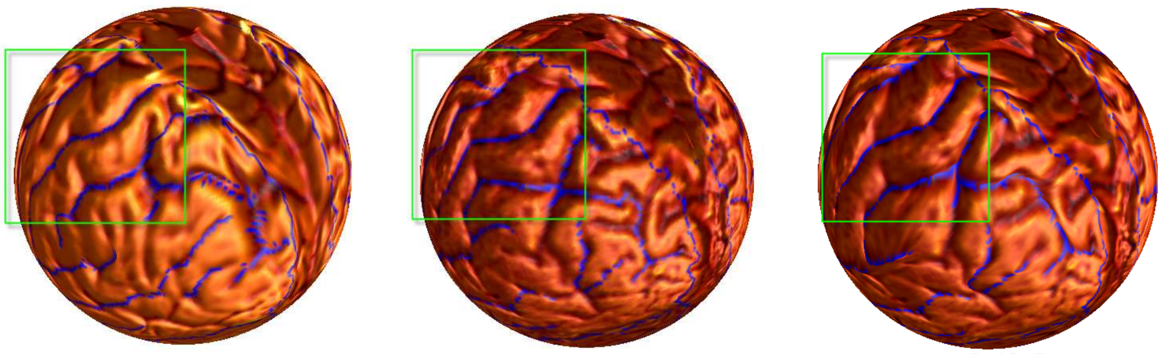
Abstract

Recent developments in brain imaging have accelerated the collection and databasing of brain maps. To compare and integrate brain data, data from multiple subjects are typically mapped into a canonical space. Surface-based approaches often map cortical surface data to a parameter domain such as a sphere or 2D plane, providing a common coordinate system for data integration. One method to do this is to conformally map cortical surfaces to the sphere. For genus zero Riemannian surfaces, harmonic maps and conformal maps are equivalent, and any genus zero Riemannian surface can be mapped conformally to a sphere. Since the cortical surface of the brain is a genus zero surface, conformal mapping offers a convenient method to parameterize cortical surfaces without angular distortion, generating an orthogonal grid on the cortex that locally preserves the metric. To compare cortical surfaces more effectively, it is advantageous to adjust the conformal parametrizations to match consistent anatomical features across subjects. This matching of cortical patterns improves the alignment of data across subjects, although it is more challenging to create a consistent conformal (orthogonal) parametrization of anatomy across subjects. Here we propose a new method, based on a new energy functional, to optimize the conformal parametrization of cortical surfaces by using landmarks.

Suppose C_1 and C_2 are the two cortical surfaces we want to compare. We proceed to compute the maps $h_1 : C_1 \to S^2$ and $h_2 : C_2 \to S^2$, which optimally retain the conformal property while matching geometric features as well as possible. This is done by minimizing the compound energy functional $E_{new} = E_{harmonic} + \lambda E_{landmark}$, where $E_{harmonic}$ is the harmonic energy of the parametrization and $E_{landmark}$ is the landmark mismatch energy. The landmark mismatch energy measures the summed Euclidean distance, in the spherical parameter domain, between the corresponding landmarks. λ is a weighting factor (Lagrange multiplier) that balances the two penalty functionals. By minimizing this energy, we sacrifice some conformality (i.e. harmonic energy is not minimized) to improve the registration of landmarks. Experimental results show that our algorithm can considerably reduce the landmark mismatch energy while effectively retaining the conformal property. In our prior work, we composed the optimal Mobius transformation with the conformal map to minimize the landmark mismatch energy. However, in that work we minimized the landmark mismatch energy only with respect to the 6 degrees of freedom in the Mobius transformation group and the map remained exactly conformal. Our new approach optimizes the energy function over all degrees of freedom in the reparametrization group. This allows the map to be non-conformal, but achieves much better landmark matching than our previous approach.

We tested our algorithm on example cortical surfaces from five subjects. Results showed that the landmark mismatch energy can be significantly reduced while effectively preserving the conformal property. And the key advantage of this approach is that any local adjustments of the mapping to match landmarks do not affect the conformality of the mapping significantly. We also examined how the parametrization changes with different weightings, λ. Results showed that the landmark matching error can be further reduced with larger λ, but conformality will be increasingly reduced. In conclusion, our algorithm provides a method to compute a map from the cortical surface of the brain to a sphere, which can effectively retain the original geometry while minimizing the landmark mismatch error between different subjects.



(a) (b) (c)

Figure 1. In (a), the cortical surface C_1 (the control) is mapped conformally ($\lambda = 0$) to the sphere. In (b), another cortical surface C_2 is mapped conformally to the sphere. Note that the sulcal landmarks appear very different from those in (a) (see landmarks in the green square). In (c), the cortical surface C_2 is mapped to the sphere using our algorithm (with $\lambda = 3$). Note that the landmarks now closely resemble those in (a) (see landmarks in the green square).

	$\lambda = 3$	$\lambda = 6$	$\lambda = 10$
$E_{harmonic}$ of the initial (conformal) parametrization:	105.0	105.0	105.0
$\lambda E_{landmark}$ of the initial (conformal) parametrization:	67.3	134.7	224.4
Initial compound energy ($E_{harmonic} + \lambda E_{landmark}$) :	172.3	239.7	329.4
Final $E_{harmonic}$	114.1 ($\nearrow$ 8.64%)	122.0 ($\nearrow$ 16.2%)	127.4 ($\nearrow$ 21.3%)
Final $\lambda E_{landmark}$	9.2 ($\searrow$ 86.2%)	11.5($\searrow$ 91.5%)	9.5 ($\searrow$ 95.8%)
Final compound energy ($E_{harmonic} + \lambda E_{landmark}$)	123.3 ($\searrow$ 28.5%)	133.4($\searrow$ 44.3%)	136.8 ($\searrow$ 58.5%)

Table 1. Numerical data from our experiment. The landmark mismatch energy is significantly reduced while the harmonic energy is only slightly increased. The table also illustrates how the results differ with different values of λ. The landmark mismatch error can be reduced by increasing λ, but conformality will increasingly be lost.