Brain Surface Parameterization with Holomorphic Differential Forms

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Abstract

Surface-based modeling is valuable in brain imaging to help analyze anatomical shape, to statistically combine or compare 3D anatomical models across subjects, and to map functional imaging parameters onto anatomical surfaces. Parameterization of these surface models involves the computation of a smooth (differentiable) one-to-one mapping of regular 2D coordinate grids onto these 3D surfaces, so that numerical quantities can be computed easily from the resulting models. Even so, it is often difficult to smoothly deform a complex 3D surface to a sphere or 2D plane without substantial angular or area distortion. Here we present a new method to parameterize brain surfaces using holomorphic differential forms. By contrast with variational approaches based on surface inflation, our method can parameterize surfaces with arbitrary complexity including branching surfaces not topologically homeomorphic to a sphere (higher-genus objects).

All orientable surfaces are Riemann surfaces and admit a global conformal structure (i.e., a parameterization that is conformal except on a set of measure zero). For high genus surfaces, a process known as holomorphic flow segmentation can be used to induce a canonical partition of the 3D surface consisting of subregions that cover the surface. In this subdivision, each surface element is either a topological disk or a cylinder and can be conformally mapped to a rectangle in the parameter domain. We compute the parameterization as follows.

Suppose S_1 is a regular surface. A map from S_1 to a local coordinate (x_1, x_2) plane is conformal when the first fundamental form is scaled by a constant For higher genus surfaces, the local conformal parameterization can be extended to cover the whole surface except at several points (zero points). By the Riemann-Roch theorem, there are $2g - 2$ zero points on a global conformal structure of a genus g closed surface. By the Circle-Valued Morse theorem, the iso-parametric curves through the zero points segment the whole surface into a set of patches, where each patch is either a topological disk, or a cylinder. The holomorphic flow segmentation is completely determined by the surface geometry and the choice of the global conformal parameterization. The resulting segmentation and parameterization are intrinsic and stable, and are continuous across the segment boundaries in the parameterization space.

In our experiments, we tested our algorithm for creating surface parameterizations of the hippocampus, lateral ventricles and cerebral cortex. Since the shape of hippocampus and lateral ventricles are not close to a sphere, the conventional method of mapping them to a sphere would require large distortions in some surface regions. To minimize this distortion, we map these surfaces to multiple rectangles. In applications that require the explicit matching of curved landmarks in the cortex, such as nonlinear matching of cortical surfaces, additional landmarks can be mapped to the rectangle boundaries. This provides a method to parameterize surfaces consistently, without recourse to intermediate representations such as inflation to a spherical domain.

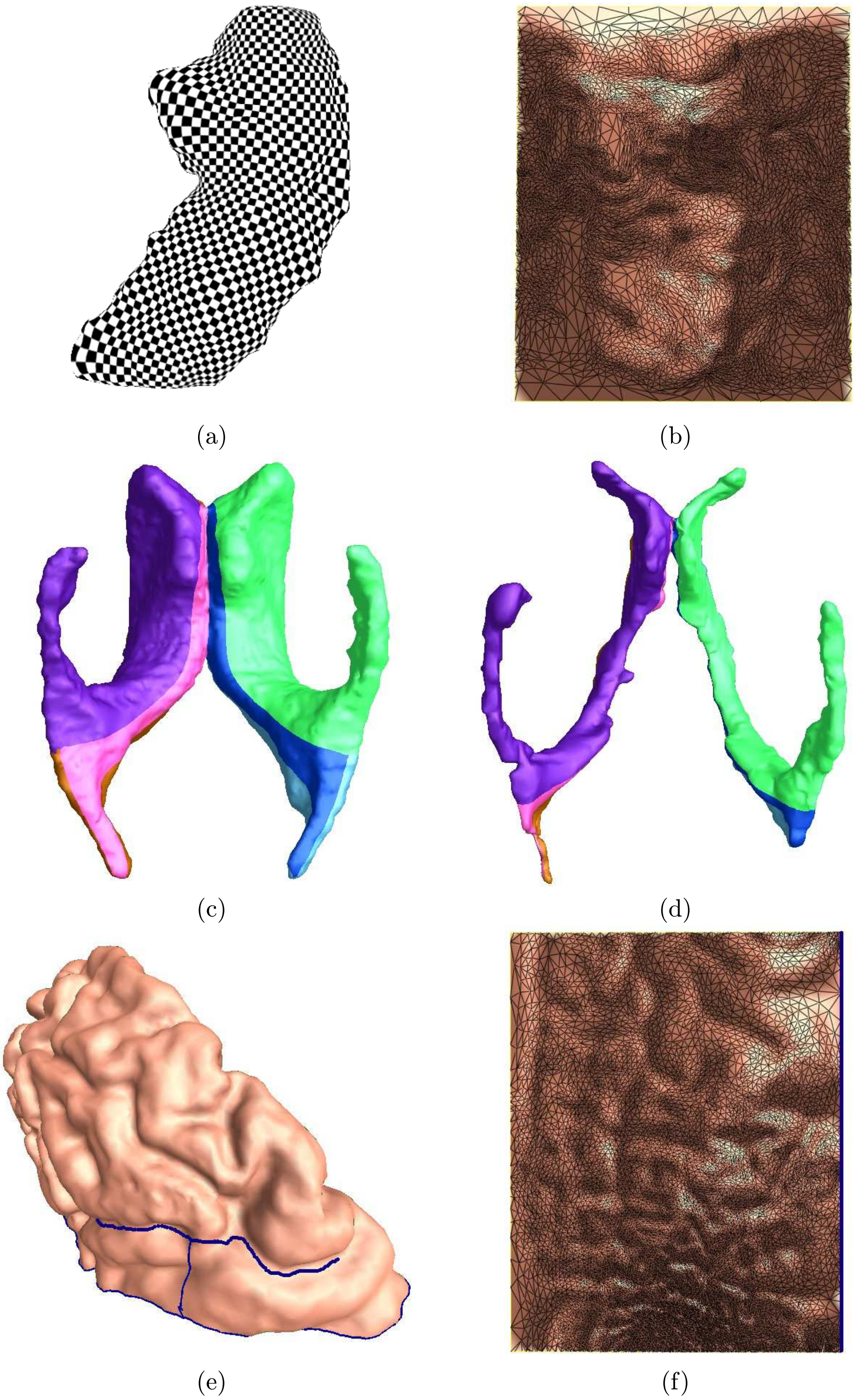

Figure 1. Illustrates parameterization results with holomorphic differential forms. (a) shows the computed global conformal structure for a hippocampal surface. (b) shows the hippocampus mapped to the rectangular parameter space. (c) is a ventricular parameterization for a 65-year-old subject with enlarged ventricles, and (d) for a 21-year-old healthy control subject, where each color denotes a separate segment in the parameterization domain. Although the two surfaces are quite different, the segmentation is consistent and could be used to nonlinearly register the two surfaces. (e) and (f) show a cortical surface segmentation (partition) in which explicit landmarks are added to provide the boundaries of the parametric patches (there are three landmarks in total and the cortical surface is segmented into four subregions). Landmark curves are indicated by thick blue lines. In (f), we can see that the landmark curve can be constrained to map to the rectangle boundary, enabling consistent parameterizations across subjects.