Volumetric Harmonic Brain Mapping using a Variational Method

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Abstract

Brain surface conformal mapping research has been successful and this motivates our more general investigation of 3D volumetric brain harmonic mapping. By transforming the full 3D brain volume to a solid sphere, our goal is to investigate how features map into this canonical 3D coordinate system in the same way as 2D conformal flattening has helped in analyzing cortical surface geometry. Nonlinear mapping of two brain volumes to a sphere may also assist with the subsequent nonlinear registration of one brain volume to another. We suggest that 3D harmonic mapping of brain volumes to a solid sphere can provide a canonical coordinate system for feature identification and segmentation, as well as anatomical normalization.

We developed two different techniques to tackle the volume mapping problem. The first finds a 3D harmonic map from a volumetric brain image to a 3D solid sphere and the second uses a sphere carving algorithm to compute a simplicial decomposition of the volume which is adapted to the surfaces.

We derived the harmonic energy computation equation. We construct a harmonic map in $\mathbb{R}^3$ *with a heat flow method. First we conformally map the boundary of the 3D volume to a sphere. During this step, we can easily include other biological constraints (e.g. mappings of superficial sulci). Our method then minimizes the volumetric harmonic energy while keeping the surface fixed. This mapping is determined by the geometric structure and also the boundary conditions. Once we fixed the mapping of the brain surfaces, the surface mapping is conformal in our case, the interior harmonic map consistently maps similar brain structures to canonical locations.*

Methods to tetrahedralize the brain for FEM analysis are somewhat rare in the literature, although they are used occasionally for surgical simulation, or mapping intraoperative brain change. In our current experiments, we apply the sphere carving algorithm to build a brain-based tetrahedral mesh. The algorithm initially constructs a large sphere that contains all the brain data. Then it keeps removing exterior tetrahedra while maintaining the surface genus number.

In our experiments, we tested our algorithm on both synthetic and brain volume data. Our synthetic data was a cube consisting of many tetrahedra. For the brain data, the input image to this algorithm is a binary 3D volumetric brain image which results from applying a Gaussian mixture tissue classifier to an MRI, in order to classify each pixel as white matter (in this case, for illustration purposes) and non-white matter. The sphere carving algorithm then builds a finite element mesh by removing tetrahedra exterior to the binary brain volume. Our experiments show that the resulting embedding can be used to induce a canonical spherical coordinate system for the brain interior. Some experimental results are shown in the Figure below.

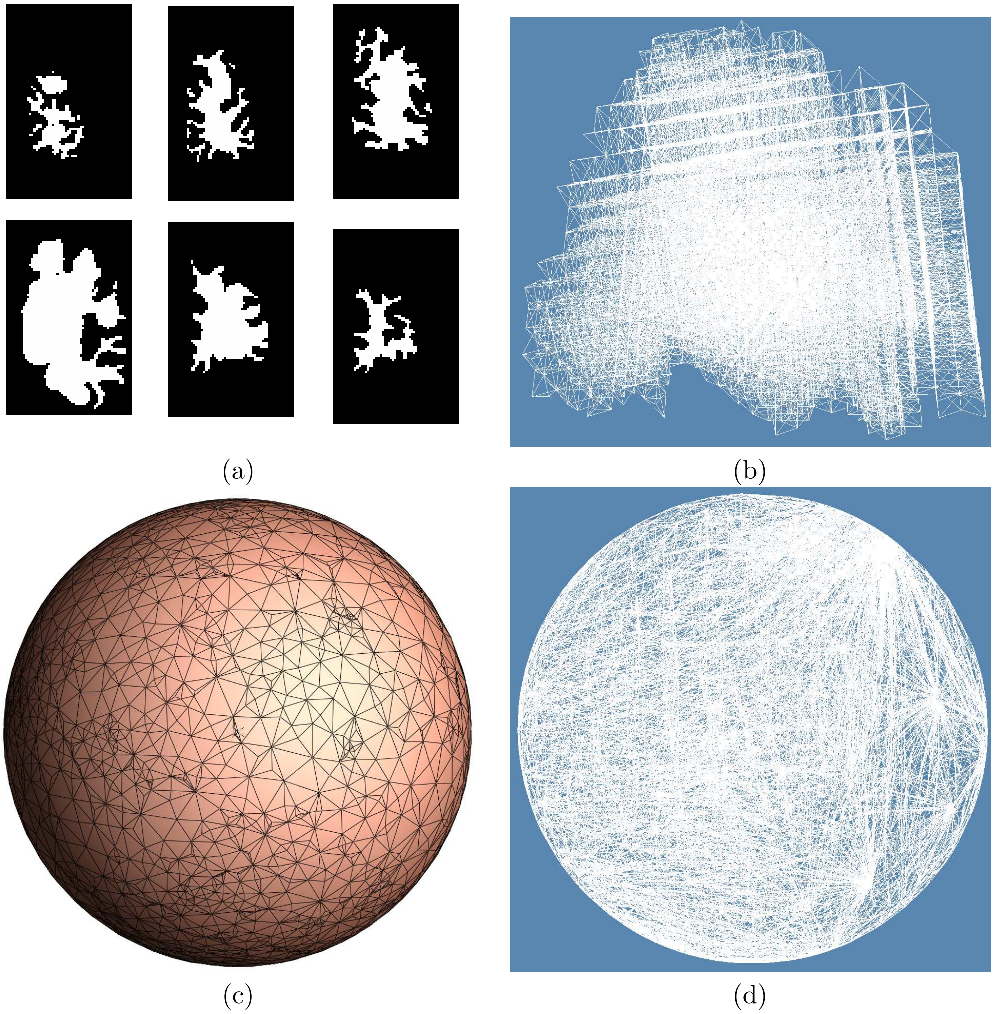

Figure 1. (a) shows some binary brain images, with white pixels are brain's white matter. (b) shows a constructed brain volume shown in wireframe. We compute a harmonic map of the brain onto a solid sphere. (c) is the surface of the solid sphere onto which there is a conformal mapping from the boundary of the brain volume. (d) is the solid sphere in wireframe mode. (d) is the solid sphere in wireframe mode. The resulting embedding can be used to induce a canonical spherical coordinate system for the brain interior.